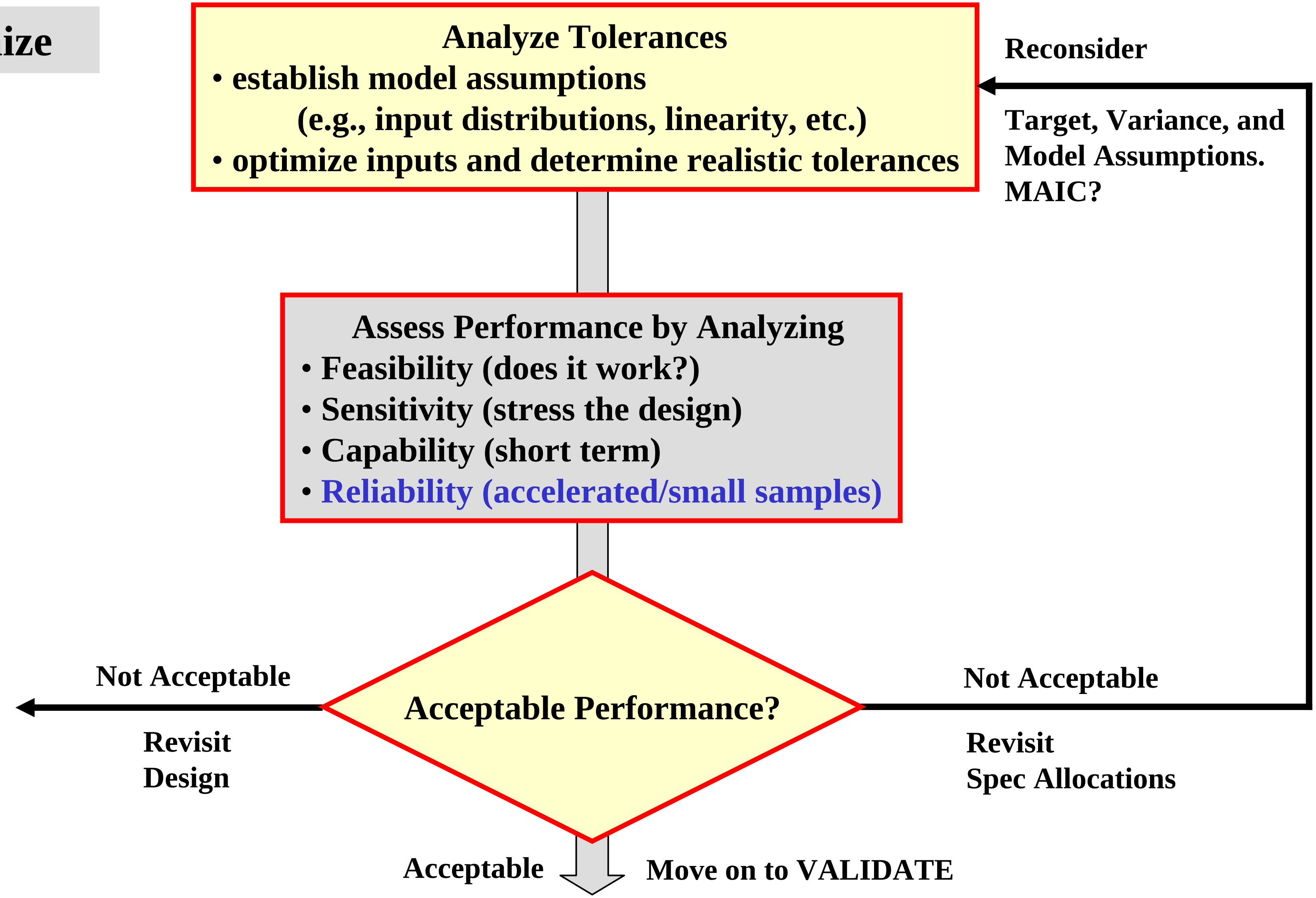
Reliability Analysis

Learning Objectives

- How to analyze Type I & Type II Censored Tests
- Introduction to some Reliability Curves
- Introduction to System Relaibility

Optimize Phase

Optimize



Reliability

Reliability :

The probability that an item will perform its intended function for a specified interval under stated environmental conditions.

Reliability — Documentation

Document	Title
MIL-STD-415	Test Provision for Electronic Systems and Associated Equipment
MIL-STD-446	Environmental Requirements for Electronic Parts
MIL-STD-470	Maintainability Program Requirements for System and Equipment
MIL-STD-471	Maintainability Demonstration
MIL-STD-690	Failure Rate Sampling Plans and Procedures
MIL-STD-721	Definitions of Effectiveness Terms for Reliability, Maintainability. Human Factors and Safety
MIL-STD-756	Reliability Prediction
MIL-STD-757	Reliability Evaluation from Demonstration Data
MIL-STD-781	Reliability Design Qualification and Production Acceptance Tests: Exponential Distribution
MIL-STD-785	Reliability Program for Systems and Equipment – Development and Production
MIL-STD-810	Environmental Test Methods
MIL-STD-1556	Government/Industry Data Exchange Program Contractor Participation Requirements
MIL-HDBK 108	Sampling Procedures and Tables for Life and Reliability Test
MIL-HDBK 109	Statistical Procedures for Determining Validity of Suppliers Attributed Inspection
MIL-HDBK 217	Reliability Prediction of Electronic Equipment

Reliability Measures

Failure Rate (λ)

– the total number of failures divided by the total number of life units (hours or cycles) expended

Mean Time Between Failures (MTBF) **for repairable items**

– the mean number of life units (hours or cycles) during which all parts of the item perform within their specified limits

Mean Time To Failures (MTTF) **for non-repairable items**

– the total number of life units (hours or cycles) of an item divided by the total number of failures

Reliability Studies

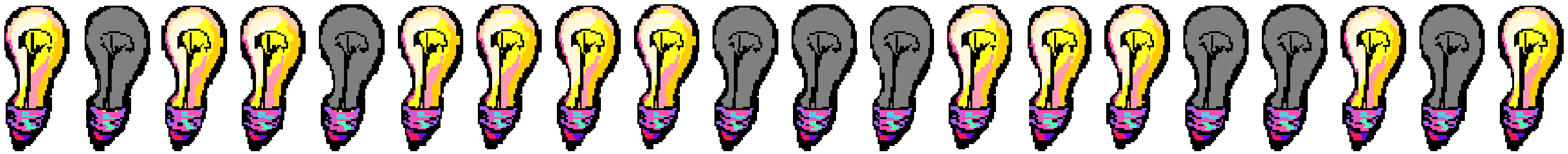
There are two common methods for reliability studies.

Type I Censoring or **Time Truncated Test**

The *n* items are placed on test for a pre-defined number of hours (or cycles). As test items fail, they are replaced.

Type II Censoring or **Failure Truncated Test**

The *n* items are placed on test. The test is truncated (or stopped) after a pre-defined number of failures *r* is observed. As items fail, they are not replaced.



Example 1

12 items are tested for up to 50,000 cycles. Failed units are repaired/replaced. During the test, 9 failures occurred.

$$
\begin{aligned}
&\text{failures observed} & r &= 9 \\
&\text{items tested} & n &= 12 \\
&\text{cycles tested} & t &= 12 \times 50{,}000 = 600{,}000 \\
&\text{failure rate} & \lambda &= 9 / 600{,}000 \\
& & &= 0.000015 \text{ items per cycle} \\
& & \text{MTBF} &= 0.000015^{-1} = 66{,}667 \text{ cycles per item}
\end{aligned}
$$

Example 1

A total of 20 items are placed on test. The test is truncated when the fourth failure occurs. The time of failure are as follows:

Failure, i	Time of Failure, t_i
1	317
2	736
3	1,032
4	1,335

$\sum t_i = 3{,}420$

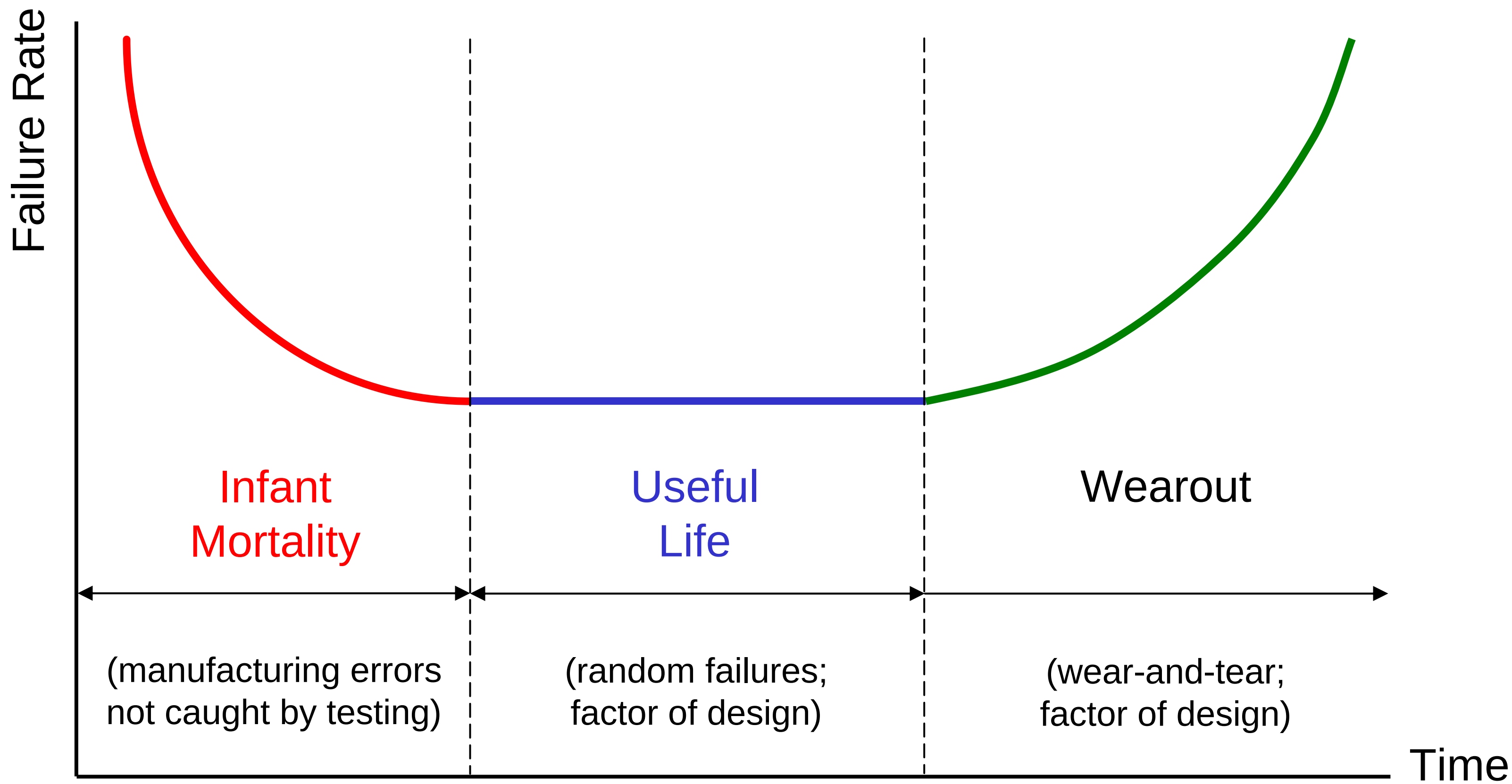
failures observed r = 4

total cycles tested t = 3,420 + 16(1,335) = 24,780

failure rate λ = 4 / 24,780 = 0.0001614 items per hour

MTBF = 0.0001614^{-1} = 6,195 hours per item

Reliability Curve



Reliability Curve

Not all components exhibit the bathtub shaped failure rate curve.

Most electronic or electrical components do not exhibit a wear-out region.

Some mechanical components may not show a constant failure rate region but exhibit a gradual transition between the early failure rate and wear-out regions.

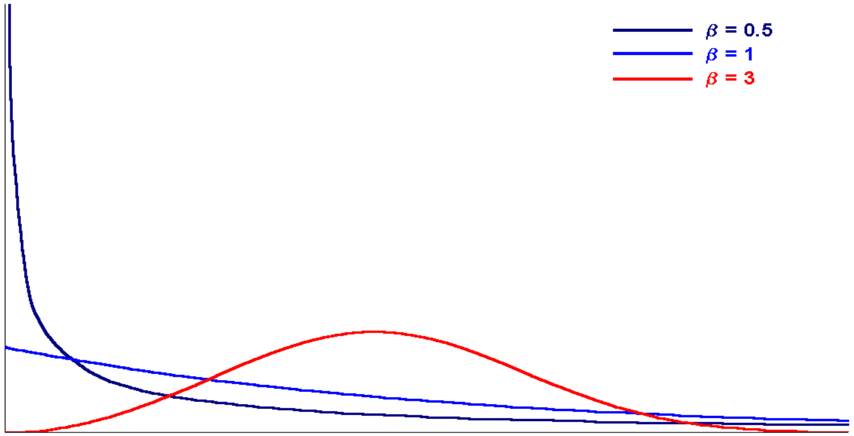
Weibull Distribution

The *Weibull distribution* is a general family of distribution with

$$Weibull(t;\beta,\theta) = \frac{\beta t^{\beta-1}}{\theta^{\beta}} e^{-\left(\frac{t}{\theta}\right)^{\beta}}$$

where *scale parameter* θ is the value at which CDF=68.17%,

and *shape parameter* β determines the shape of the PDF.



β	Distribution
1	Exponential
2	Rayleigh
3.43927	Normal

Weibull Distribution

The Weibull distribution is commonly used in reliability studies because it can model the various stages of the reliability curve by changing its shape parameter β.

Stage	Failure Rate	Shape Parameter
Infant Mortality	Decreasing	$\beta < 1$
Useful Life	Constant	$\beta = 1$
Wearout	Increasing	$\beta > 1$

Other Reliability Probability Models

Distribution	To Model Applications
Extreme Value	hazard rate is initially constant and then increases rapidly with time
Normal	mechanical components are subjected to repeated cyclic loads
Log-Normal	single semiconductor failure mechanisms, accelerated life tests
Gamma	failures that take place in *k* stages
Beta	components with a finite interval of life

System Reliability

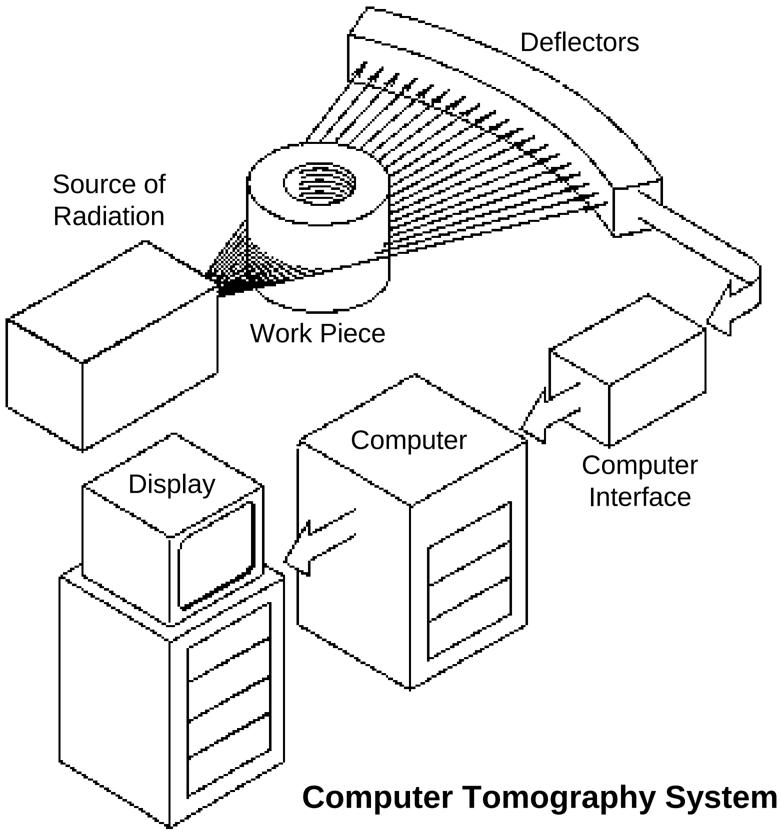
A *system* (a product or service) is a collection of components arranged according to a specific design in order to achieve desired functions with acceptable performance and reliability measures.

System reliability needs to be evaluated as many times as the design changes.

System Reliability

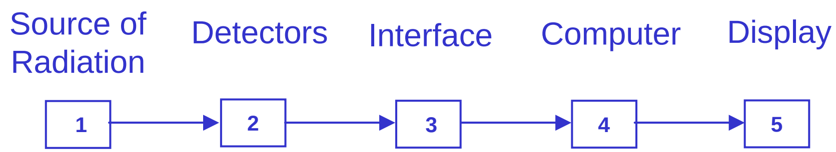
- Construct a *reliability block diagram*
  - a graphical representation of the components in a system and how they are connected
- Create a *reliability graph*
  - a line representation of the blocks that indicates the path

Example 4 — Computer Tomography System

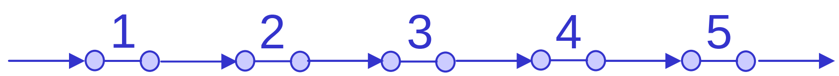


Computer Tomography System

Reliability Block Diagram



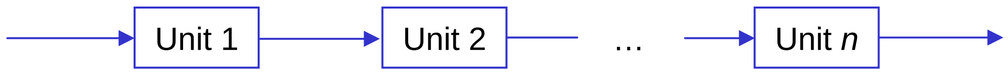
Reliability Graph



A system or a sub-system can be analyzed at different levels down to the component level.

System Reliability — Series System

A *series system* is composed of n components (sub-systems) connected in series.



A failure of any component will result in failure of the entire system.

Reliability of the system is the probability that all components are operational.

System Reliability — Series System

p_i = probability that the i^{th} unit is operational
R = reliability of the system

R = probability that all n units are operational

$$R = \prod_i^n p_i = p_1 \, p_2 \cdots p_n$$

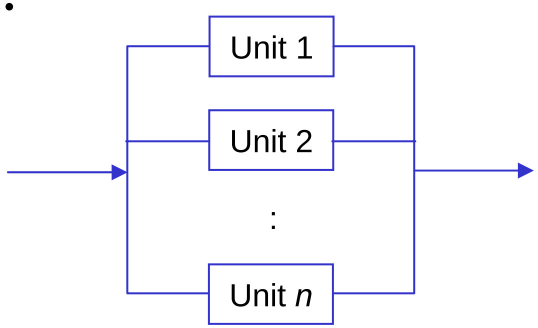
Example 5

A series system consists of three components and the probabilities that components 1, 2 and 3 being operational are 0.9, 0.8 and 0.75 respectively.

$$\text{Reliability of System} \quad R = \prod_{i=1}^{n} p_i = p_1\, p_2\, p_3$$
$$= (0.9)(0.8)(0.75)$$
$$= 0.54$$

System Reliability — Parallel System

In a *parallel system*, the *n* components (sub-systems) are connected in parallel.



The failure of one or more paths still allows the remaining path(s) to perform properly.

Reliability is the probability that any one path is operational.

System Reliability — Parallel System

p_i = probability that the i^{th} unit is operational

R = reliability of the system

R = 1 – probability that all n units are not operational

$$R = 1 - \prod_i^n (1 - p_i) = 1 - \left[(1 - p_1)(1 - p_2)\cdots(1 - p_n)\right]$$

Example 6

A system consists of three components in parallel. The probabilities of the three components 1,2 and 3 being operational are 0.9, 0.8 and 0.75 respectively.

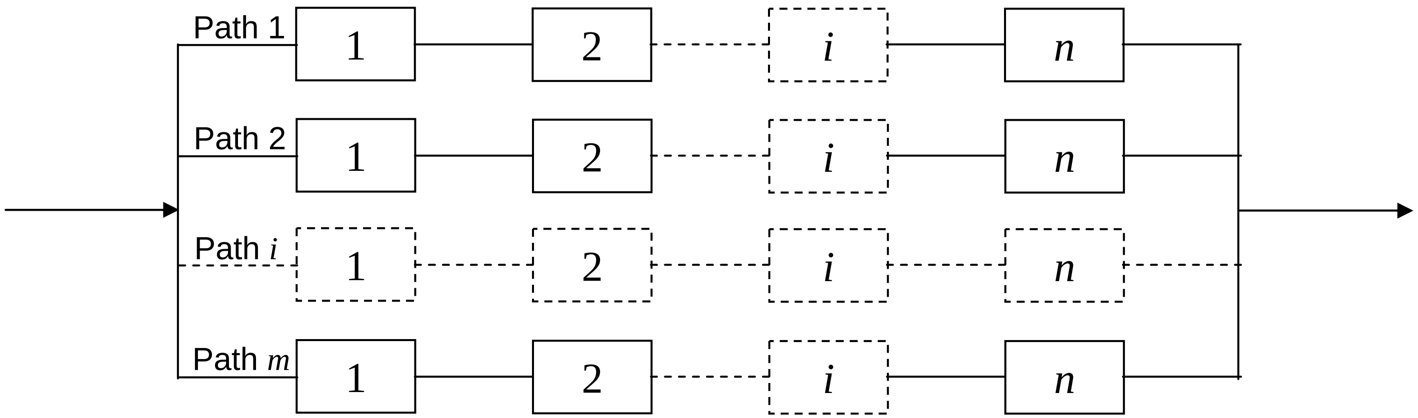
$$
\begin{aligned}
\text{Reliability of System } R &= 1-\prod_i^n (1-p_i) \\
&= 1-[(1-p_1)(1-p_2)\cdots(1-p_n)] \\
&= 1-[(1-0.9)(1-0.8)\cdots(1-0.75)] \\
&= 0.995
\end{aligned}
$$

System Reliability — Mixed System

a) Parallel-Series

Consists of *m* parallel paths.

Each path has *n* units connected in series

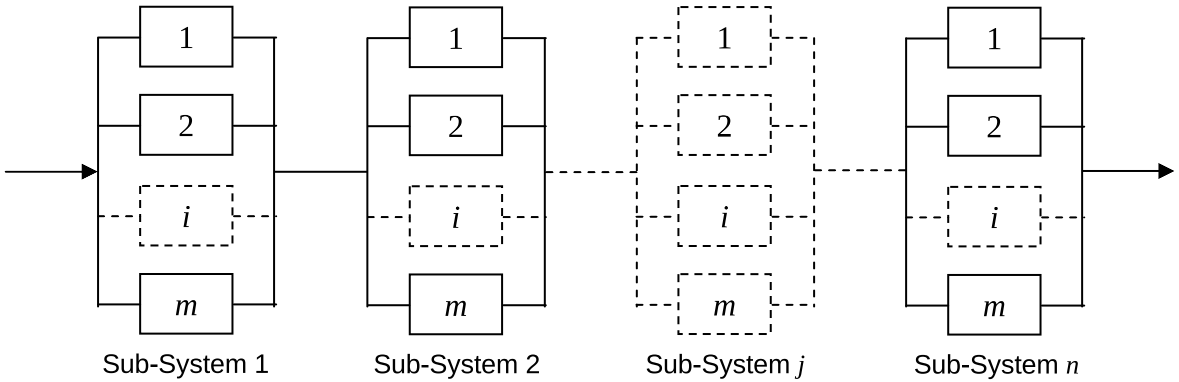


System Reliability — Mixed System

b) Series-Parallel

Consists of *n* sub-systems in series.

Each sub-system has *m* units connected in parallel.

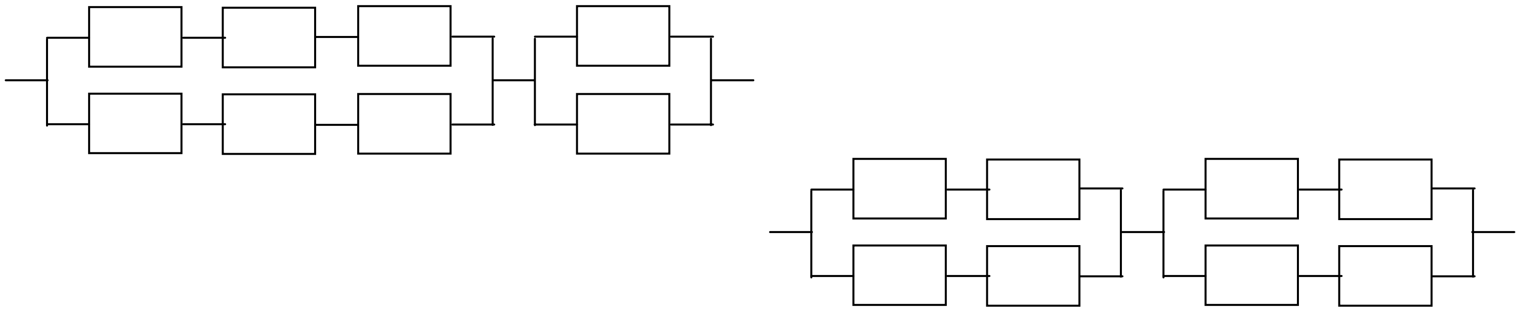


In general, series-parallel systems have higher reliabilities than parallel-series systems when both have an equal number of units and each unit has the same probability of operation.

System Reliability — Mixed System

c) Mixed-Parallel

No specific arrangement of units, other than the fact that units are connected in parallel and series configurations.

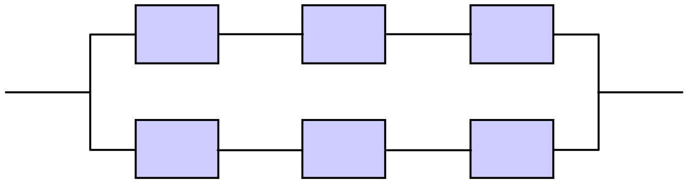


Reliability of a mixed-parallel system can be estimated by using the basic equations for series and parallel systems.

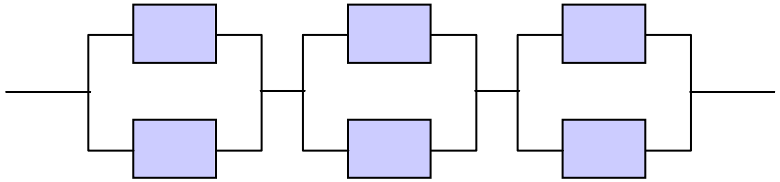
Example 7

Six identical units each having a reliability of 0.85 are arranged in parallel-series, series-parallel and mixed-parallel configurations.

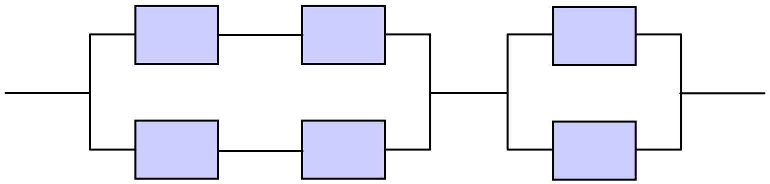
Determine their respective system reliabilities.



(a) Parallel-Series



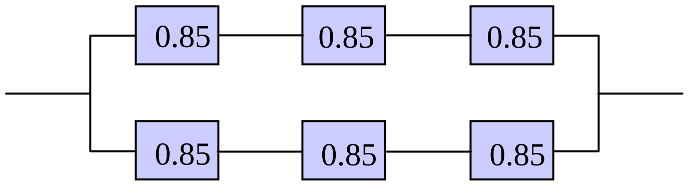
b) Series-Parallel



c) Mixed-Parallel

Example 7.1

a) Parallel- Series System



For each path *i*,

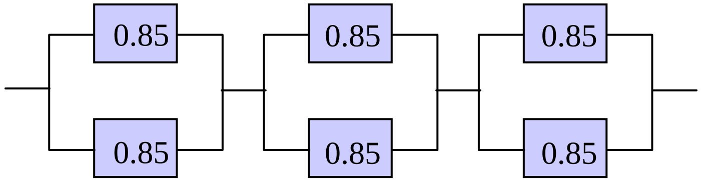
$$R_i = p \cdot p \cdot p$$
$$= 0.85^3$$

For the system,

$$R_s = 1 - (1 - R_1)(1 - R_2) = 1 - (1 - 0.85^3)(1 - 0.85^3)$$
$$= 0.8511$$

Example 7.2

b) Series-Parallel System



For each sub-system j,

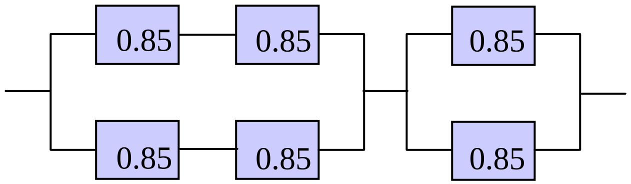
$$
\begin{aligned}
R_j &= 1-(1-p)(1-p) \\
&= 1-(1-0.85)(1-0.85) \\
&= 0.9775
\end{aligned}
$$

For the system,

$$
\begin{aligned}
R_s &= R_1 R_2 R_3 = (0.9775)(0.9775)(0.9775) \\
&= 0.9340
\end{aligned}
$$

Example 7.3

c) Mixed-Parallel System



For sub-system 1,

$$
\begin{aligned}
R_1 &= 1-\left[1-\left(1-p^2\right)^2\right] \\
&= 1-\left[1-\left(1-0.85^2\right)^2\right] \\
&= 0.9230
\end{aligned}
$$

For the system,

$$
\begin{aligned}
R_s &= R_1 R_2 \\
&= (0.9230)\,(0.9774) \\
&= 0.9022
\end{aligned}
$$

For sub-system 2,

$$
\begin{aligned}
R_2 &= 1-(1-p)(1-p) \\
&= 1-(1-0.85)(1-0.85) \\
&= 0.9775
\end{aligned}
$$

System Reliability — Redundancy

Series systems do not offer high level of confidence in high risk situations.

Parallel systems offers high confidence through redundancy:

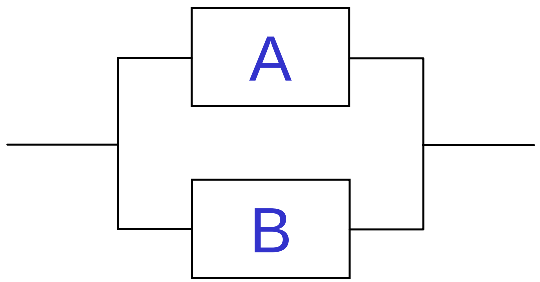
a) active redundancy

b) standby redundancy

Higher confidence in the probability of success is attained at the expense of increased cost, weight, size, maintenance, etc.

Active Redundancy

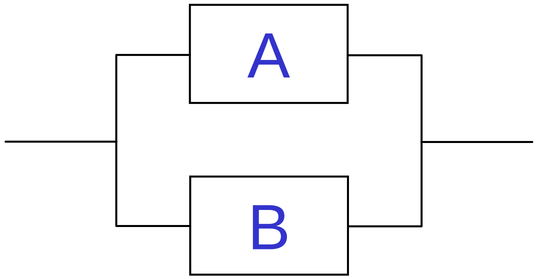
Both components are active or functioning during the entire life of the system.



A	B	System
✓	✓	functional
✓	×	functional
×	✓	functional
×	×	failure

Example 8

<u>Component</u>	<u>Reliability</u>
A	0.99
B	0.95

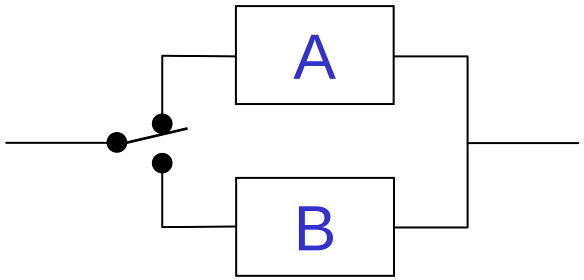


Reliability of System

$$
\begin{aligned}
R &= 1-\prod_i^n\left(1-p_i\right)\\
&= 1-\left(1-p_A\right)\left(1-p_B\right)\\
&= 1-\left(1-0.99\right)\left(1-0.95\right)\\
&= 1-\left(0.01\right)\left(0.05\right)\\
&= 1-0.0005\\
&= 0.9995
\end{aligned}
$$

Standby Redundancy

The unit on standby is usually considered dormant with zero probability of failure.



(standby unit)

Switching between the components may be

- perfect, i.e. $p_{s/w} = 1$
- imperfect, i.e. $p_{s/w} < 1$

Standby Redundancy

1. Equal Failure Rates — Perfect Switching

$$R_t = e^{-\lambda t} + \lambda t e^{-\lambda t} = e^{-\lambda t}\left(1 + \lambda t\right)$$

2. Unequal Failure Rates — Perfect Switching

$$R_t = e^{-\lambda_1 t} + \frac{\lambda_1}{\lambda_1 + \lambda_2}\left(e^{-\lambda_1 t} - e^{-\lambda_2 t}\right)$$

3. Equal Failure Rates — Imperfect Switching

$$R_t = e^{-\lambda t}\left(1 + R_{s/w}\lambda t\right)$$

4. Unequal Failure Rates — Imperfect Switching

$$R_t = e^{-\lambda_1 t} + R_{s/w}\frac{\lambda_1}{\lambda_1 + \lambda_2}\left(e^{-\lambda_1 t} - e^{-\lambda_2 t}\right)$$

Summary

- Censored data provides valuable information.
- System reliability should be duly considered when designing a system of interacting components.

End of Module